DATA PROCESSING AND ANALYSIS FOR ABLE SURFACE CHARACTERIZATION, EDDY CORRELATION FLUX MEASUREMENT SYSTEM

BACKGROUND

1. Definition of statistical terms

The mean of a time series of n data values is the average value of the variable:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{1}$$

The variance is the average square of the departure of the variable from its mean:

$$\sigma_x{}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \overline{x}^2 \tag{2}$$

The covariance of two time series, each with n data values, is the average product of the departures of the two variables from their respective means:

$$\sigma_{xy} = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) = \frac{1}{n}\sum_{i=1}^{n} x_i\, y_i - \overline{x}\,\overline{y} \tag{3}$$

Often sensors are used whose signal output lags the physical sampling, for example, air chemistry sensors. In order to properly compute the covariances with the output signals from these sensors, the output signals from the other sensors must be correspondingly lagged.

2. Running mean removal

The eddy correlation technique is based on the assumption that conditions are steady, i.e., that there are no low frequency trends in the data. In nature, steady state conditions rarely exist. It is common practice to detrend the data by using departures from a short term (or "running") mean. The appropriate low frequency "cutoff" is primarily a function of the mean wind speed and the height above the ground. For instrumentation deployed at the height of 3 meters, variances and covariances are computed for departures from the mean of the past 200 seconds. The "running" means are determined recursively:

$$\langle x \rangle_i = \left(1 - e^{-\Delta t/\tau}\right) x_i + \left(e^{-\Delta t/\tau}\right)\langle x \rangle_{i-1} \tag{4}$$

where Δt is the sampling interval and τ is the time constant.

3. Coordinate rotations

Coordinate rotation transforms, which result in zero vertical and transverse mean wind speeds, are applied to the variances and covariances from the running means before the fluxes are computed.

The angles θ and φ are defined:

$$\theta = \arctan\left(\bar{v}/\bar{u}\right) \qquad \text{or} \qquad \theta = \arcsin\left(\bar{v}/\sqrt{\bar{u}^2+\bar{v}^2}\right) \tag{5}$$

$$\varphi = \arctan\left(\frac{\bar{w}}{\sqrt{\bar{u}^2+\bar{v}^2}}\right) \qquad \text{or} \qquad \varphi = \arcsin\left(\frac{\bar{w}}{\sqrt{\bar{u}^2+\bar{v}^2+\bar{w}^2}}\right) \tag{6}$$

The rotated means are defined:

$$\bar{\hat{u}} = \bar{u}\cos\theta\cos\varphi + \bar{v}\sin\theta\cos\varphi + \bar{w}\sin\varphi \tag{7}$$

$$\bar{\hat{v}} = \bar{v}\cos\theta - \bar{u}\sin\theta \tag{8}$$

$$\bar{\hat{w}} = \bar{w}\cos\varphi - \bar{u}\cos\theta\sin\varphi - \bar{v}\sin\theta\sin\varphi \tag{9}$$

The rotated variances are defined:

$$\begin{aligned}\sigma_{\hat{u}}^2 = {} & \sigma_u^2\cos^2\theta\cos^2\varphi + \sigma_v^2\sin^2\theta\cos^2\varphi + \sigma_w^2\sin^2\varphi \\ & +2\sigma_{uv}\sin\theta\cos\theta\cos^2\varphi + 2\sigma_{uw}\cos\theta\sin\varphi\cos\varphi \\ & +2\sigma_{vw}\sin\theta\sin\varphi\cos\varphi\end{aligned} \tag{10}$$

$$\sigma_{\hat{v}}^2 = \sigma_v^2\cos^2\theta + \sigma_u^2\sin^2\theta - 2\sigma_{uv}\sin\theta\cos\theta \tag{11}$$

$$\begin{aligned}\sigma_{\hat{w}}^2 = {} & \sigma_w^2\cos^2\varphi + \sigma_u^2\cos^2\theta\sin^2\varphi + \sigma_v^2\sin^2\theta\sin^2\varphi \\ & -2\sigma_{uw}\cos\theta\sin\varphi\cos\varphi - 2\sigma_{vw}\sin\theta\sin\varphi\cos\varphi \\ & +2\sigma_{uv}\sin\theta\cos\theta\sin^2\varphi\end{aligned} \tag{12}$$

The rotated covariances are defined:

$$\sigma_{\hat{u}\hat{w}} = \sigma_{uw}\cos\theta\left(\cos^2\varphi - \sin^2\varphi\right) + \sigma_{vw}\sin\theta\left(\cos^2\varphi - \sin^2\varphi\right)$$
$$-2\sigma_{uv}\sin\theta\cos\theta\sin\varphi\cos\varphi - \sigma_u^2\cos^2\theta\sin\varphi\cos\varphi$$
$$-\sigma_v^2\sin^2\theta\sin\varphi\cos\varphi + \sigma_w^2\sin\varphi\cos\varphi \qquad (13)$$

$$\sigma_{\hat{u}\hat{v}} = \sigma_{uv}\cos\varphi\left(\cos^2\theta - \sin^2\theta\right) + \sigma_{vw}\cos\theta\sin\varphi$$
$$-\sigma_{uw}\sin\theta\sin\varphi + \left(\sigma_v^2 - \sigma_u^2\right)\sin\theta\cos\theta\cos\varphi \qquad (14)$$

$$\sigma_{\hat{v}\hat{w}} = \sigma_{vw}\cos\theta\cos\varphi - \sigma_{uw}\sin\theta\cos\varphi$$
$$-\sigma_{uv}\sin\varphi\left(\cos^2\theta - \sin^2\theta\right) + \left(\sigma_u^2 - \sigma_v^2\right)\sin\theta\cos\theta\sin\varphi \qquad (15)$$

and

$$\sigma_{\hat{w}s} = \sigma_{ws}\cos\varphi - \sigma_{us}\cos\theta\sin\varphi - \sigma_{vs}\sin\theta\sin\varphi \qquad (16)$$

$$\sigma_{\hat{u}s} = \sigma_{us}\cos\theta\cos\varphi + \sigma_{vs}\sin\theta\cos\varphi + \sigma_{ws}\sin\varphi \qquad (17)$$

$$\sigma_{\hat{v}s} = \sigma_{vs}\cos\theta - \sigma_{us}\sin\theta \qquad (18)$$

where s is any scalar quantity, e.g., virtual temperature, water vapor density, or CO_2 density.

4. Fluxes

The vertical fluxes of momentum, sensible heat, latent heat, and CO_2 can be obtained from the covariances of the vertical wind with the horizontal wind, the vertical wind with temperature, the vertical wind with water vapor, and the vertical wind with CO_2, respectively. Momentum flux is rarely listed explicitly. Instead, the friction velocity is computed by:

$$u_* = \sqrt{-\sigma_{uw}} \qquad (19a)$$

Sensible heat flux is determined by:

$$H = -\rho c_p \sigma_{wT} \qquad (20a)$$

latent heat flux is determined by:

$$L_v E = -\rho L_v \sigma_{wq} \tag{21a}$$

and CO_2 flux is the covariance:

$$\sigma_{wCO_2} \tag{22}$$

where ρ is air density, c_p is the specific heat of air at constant pressure, L_v is the latent heat of vaporization of water, and q is specific humidity. The constants ρ, c_p, and L_v are functions of water vapor density (ρ_v), ambient air temperature (T_a), and atmospheric pressure (p). The mixing ratio is calculated from mean values of ρ_v (in g/m^3), T_a (in °K), and p (in millibars):

$$r = \frac{2.87x10^{-3}\,\overline{\rho_v}\,\overline{T_a}}{\bar{p}} \tag{23}$$

The air density in g/m^3 is computed from the mixing ratio and mean values of T_a (in °K) and p (in millibars):

$$\rho = \frac{\bar{p}(1+r)}{2.87x10^{-3}\,\overline{T_a}(1+1.6078r)} \tag{24}$$

The specific heat of dry air at constant pressure in Watt·sec/g·°K is corrected for the presence of water vapor:

$$c_p = 1.006 + 1.846r \tag{25}$$

and the latent heat of vaporization of water in Watt·sec/g is corrected for air temperature effects:

$$L_v = 2501.3 - 2.366\overline{T_a} \tag{26}$$

where T_a is expressed in °C.

5. Spectral analysis

The Fourier Transform is a mathematical technique used to transform a time series from the time domain to the frequency domain. It is expressed in integral form as:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \tag{27}$$

where $j = \sqrt{-1}$ and $\omega = 2\pi f$, f being frequency in Hz. The Fourier coefficients, $X(\omega)$, are complex valued. Numerous algorithms exist to perform a Fast Fourier Transform (FFT), which minimize the number of computations. For discrete FFT's, ω varies from $-\omega_N$ to ω_N (± the Nyquist frequency $\omega_N = \pi/\Delta t$) with steps of $\Delta\omega = 2\pi/n\Delta t$ where Δt is the sampling interval and n is the number of samples.

The power spectrum is used to view the energy characteristics (variance) of the signal in the frequency domain. The integral of the power spectrum is equal to the variance of the input. The two-sided power spectrum can be obtained from the Fourier coefficients:

$$S_x(\omega) = \frac{2\pi}{\Delta\omega\, n^2} X(\omega) X^*(\omega) \tag{28}$$

where * denotes the complex conjugate. The power spectrum is always real valued (not complex). A one-sided power spectrum is obtained by summing the estimates for ω and -ω:

$$P_x(\omega) = \frac{2\pi}{\Delta\omega\, n^2}\left(X(\omega)\, X^*(\omega) + X(-\omega)\, X^*(-\omega)\right) \qquad (29a)$$

Since the input data are all real-valued, symmetry allows the simplification:

$$P_x(\omega) = \frac{4\pi}{\Delta\omega\, n^2}\left(X(\omega) X^*(\omega)\right) \qquad (29b)$$

The cross spectrum is used to view the correlation between two signals in the frequency domain. The two-sided cross spectrum can be obtained from the two sets of Fourier coefficients:

$$S_{xy}(\omega) = \frac{2\pi}{\Delta\omega\, n^2}\; X(\omega)\, Y^*(\omega) \qquad (30)$$

A one-sided cross spectrum is obtained by summing the estimates of the cross spectrum for ω and -ω:

$$\begin{aligned} G_{xy}(\omega) &= C_{xy}(\omega) - i\, Q_{xy}(\omega) \\ &= \frac{2\pi}{\Delta\omega\, n^2}\left(X(\omega)\, Y^*(\omega) + X(-\omega)\, Y^*(-\omega)\right) \end{aligned} \qquad (31a)$$

Since both input data series are real-valued, symmetry allows the simplification:

$$G_{xy}(\omega) = \frac{4\pi}{\Delta\omega\, n^2}\left(X(\omega) Y^*(\omega)\right) \qquad (31b)$$

The co-spectrum is the real portion of the one-sided cross spectrum and the quadrature spectrum is the negative of the imaginary portion of the one-sided cross spectrum. The integral of the co-spectrum is equal to the covariance of the inputs. The ratio of the quadrature to the co-spectrum can be used to determine the phase relation between the inputs.

The shapes of power spectra and co-spectra are well known for different atmospheric conditions. Departures of the spectra from normal are indicative of instrumentation problems or a divergence from typical atmospheric conditions. Thus the spectra can be used as a quality check on the fluxes.

SURFACE CHARACTERIZATION STATION DATA PROCESSING AND ANALYSIS

Data are processed over 30 minute periods, which start exactly on the hour or half-hour. All raw u, v, w, T_v, ρ_v, and CO_2 data (36,000 points each) are written to a file each half hour. The first 32k (32,768) values are analyzed.

Means of u, v, w, T_v, ρ_v, CO_2, and s are computed (Eq. 1). Variances of departures from the means of u, v, w, T_v, ρ_v, CO_2, and s, and covariances of departures from the means of uv, uw, uT_v, $u\rho_v$, uCO_2, us, vw, vT_v, $v\rho_v$, vCO_2, vs, wT_v, $w\rho_v$, wCO_2, ws, $T_v\rho_v$, $T_v\,CO_2$, $T_v s$, $\rho_v CO_2$, $\rho_v s$, and $CO_2 s$ are computed (Eq. 2 and 3). The means of the departures from the "running" means are computed (Eq. 1) for u, v, w, T_v, ρ_v, CO_2, and s. Variances of departures from the "running" means of u, v, w, T_v, ρ_v, CO_2, and s and covariances of departures from the "running" means of uv, uw, uT_v, $u\rho_v$, uCO_2, us, vw, vT_v, $v\rho_v$, vCO_2, vs, wT_v, $w\rho_v$, wCO_2, ws, $T_v\rho_v$, $T_v\,CO_2$, $T_v s$, $\rho_v CO_2$, $\rho_v s$, and $CO_2 s$ are computed (also Eq. 2 and 3).

Three dimensional coordinate rotations, calculated using Eqs. 5 through 18, are applied to the variances of departures from the "running" means of u, v, and w and covariances of departures from the "running" means of uv, uw, uT_v, $u\rho_v$, uCO_2, vw, vT_v, $v\rho_v$, vCO_2, wT_v, $w\rho_v$ and wCO_2.

A vector averaged wind speed and direction are calculated from the means of u and v by:

$$\text{wind\_speed} = \sqrt{\bar{u}^2 + \bar{v}^2} \tag{32}$$

$$\text{wind\_dir} = \arctan\left(\bar{v}/\bar{u}\right) + \text{boom\_angle} \qquad \text{or}$$

$$\text{wind\_dir} = \arcsin\left(\bar{v}\Big/\sqrt{\bar{u}^2 + \bar{v}^2}\right) + \text{boom\_angle} \tag{33}$$

where u, v, and wind_speed are expressed in m/s and wind_dir, the arctan, and boom_angle are expressed in degrees.

An estimate of the standard deviation of wind direction is calculated from the vector averaged wind speed and the mean value of s by using the algorithm:

$$\sigma_\theta = 81\sqrt{1 - \frac{\text{wind\_speed}}{\bar{s}}} \tag{34}$$

where wind_speed and s are expressed in m/s.

The mixing ratio, air density, specific heat of dry air at constant pressure, and the latent heat of vaporization of water are computed (Eqs. 23-26) from the average values of water vapor density, air temperature, and barometric pressure obtained from data from a colocated Automatic Weather Station.

Finally, the friction velocity and the vertical fluxes of sensible heat, latent heat, and CO_2 are calculated from the coordinate rotated covariances:

$$u_* = \sqrt{-\sigma_{uw}} \tag{19b}$$

$$H_v = -\rho\, c_p\, \sigma_{wT_v} \tag{20b}$$

$$L_v E = -L_v\, \sigma_{w\rho_v} \tag{21b}$$

The latter equations differ from Eq. 20a and 21a because the sensors measure sonic temperature and water vapor density.

Power and co-spectra are computed for quality control and assurance of the data. An in-place, direct, Fast Fourier Transform (FFT) of two arrays, one real and the other imaginary, is used to transform the u, v, w, T_v, ρ_v , and CO_2 data from the time domain to the frequency domain. A two-butterfly, Cooley-Tukey, radix-2 FFT and a lookup table for the sine - cosine transfer functions are used. The FFT returns the Fourier coefficients for the cosine transform in the real array and for the sine transform in the imaginary array. Two routines are used to produce the Fourier coefficients. The first uses the symmetry of the transform of a real-valued input:

$$X(-\omega) = X^*(\omega) \tag{35}$$

and of the transform of purely imaginary input:

$$Y(-\omega) = -Y^*(\omega) \tag{36}$$

where * denotes complex conjugation. By putting one time series in the real array and a second series in the imaginary array, performing the FFT, and then unpacking the resulting arrays into arrays for the two series, two transforms are accomplished by a single FFT. The second routine, used when the number of inputs is an odd number, puts the input series into the real array and zero's the imaginary array. Both routines return the coefficients for frequency $i\Delta\omega$ in the i'th location, for all i = 0, 1, 2, ..., (n/2)-1, where n is the number of data points, $\Delta\omega = 2\pi/n\Delta t$ is the frequency step, and Δt is the sampling interval. No coefficients are returned for $(n/2)\Delta\omega$.

Raw power spectra for u, v, w, T_v, ρ_v , and CO_2 are computed from the Fourier coefficients (Eq. 29b) by summing the squares of the real and the imaginary coefficients:

$$S_x(i) = \frac{4\pi}{\Delta\omega n^2}\left(R_x^2(i+1) + I_x^2(i+1)\right) \tag{37}$$

for i = 0, 1, 2, ..., (n/2)-2. The spectral estimate for frequency of zero is not computed.

The raw power spectra are smoothed and the number of estimates reduced by averaging. The first (lowest frequency) estimate is not averaged. The second and third are averaged to produce the second value of the smoothed spectra. The fourth through the seventh are averaged to produce the third value of the smoothed spectra. The next eight, then sixteen, and then thirty-two are averaged to produce the fourth, fifth, and sixth values of the smoothed spectra. The remaining are averaged over sixty-four estimates each to produce the seventh through the one hundred and first value. Each of the smoothed spectral values is multiplied by the square of the calibration slope and the frequency in the center of the respective averaging interval. The frequencies are retrieved from a lookup table.

Raw co-spectra for uv, uw, uT_v, $u\rho_v$, uCO_2, vw, vT_v, $v\rho_v$, vCO_2, wT_v, $w\rho_v$, wCO_2, $T\rho_v$, TCO_2, and $\rho_v CO_2$ are computed from the Fourier coefficients (Eq. 31b) by summing the products of the real coefficients and the products of the imaginary coefficients for the two variables:

$$G_{xy}(i) = \frac{4\pi}{\Delta\omega n^2}\left(R_x(i+1)R_y(i+1) + I_x(i+1)I_y(i+1)\right) \qquad (38)$$

for i = 0, 1, 2, ..., (n/2)-2. The co-spectral estimate for frequency of zero is not computed.

The raw co-spectra are smoothed and the number of estimates reduced by averaging in the same manner as the power spectra. Each of the smoothed spectral values is multiplied by the product of the calibration slopes and the frequency in the center of the respective averaging interval.